Curriculum Vitae et Studiorum

Surname Cordella

Name Davide

Positions

1/1/2023 - 31/12/2023 Post-Doc Position (Assegno di ricerca), Università degli Studi di Ferrara: *Analisi e Geometria Ipercomplessa* - (en) *Hypercomplex Analysis and Geometry*

1/3/2022 - 31/12/2022 Post-Doc Position (Assegno di ricerca), Università di Roma 'Tor Vergata': *Semigruppi di Funzioni Olomorfe (MUR - Fondo Dipartimenti di Eccellenza - CUP: E83C18000100006)* - (en) *Semigroups of Holomorphic Functions*

Visiting

January 2023 IMUS - Instituto de Matemáticas Universidad de Sevilla (Spain)

Education

9/12/2021 Ph.D. in Mathematics at Università di Roma 'Tor Vergata'

Thesis: *On speeds of convergence of non-elliptic holomorphic semigroups in the unit disc* (Advisor: prof. Filippo Bracci)

23/3/2018 Master's Degree (*Laurea Magistrale*) in Mathematics at Alma Mater Studiorum – Università di Bologna, 110/110 cum laude

Thesis: *Equilibrium measures on trees and square tilings* (Advisor: prof. Nicola Arcozzi)

30/10/2015 Degree in Mathematics at Alma Mater Studiorum – Università di Bologna, 110/110 cum laude

Thesis: *Funzioni a variazione limitata per la ricolorazione di un'immagine* (Advisor: prof. Giovanna Citti)

Research

Current work

During the year in Università di Ferrara, I started working and getting interested in questions about hypercomplex analysis (in particular the theory of slice regular functions over the division ring of quaternions $\mathbb{H}$). In this setting it is established a kind of Schwarz-Pick Lemma for regular maps from the unit ball to itself. As we know that the complex analogous leads to many generalizations and several estimates and deep results in geometric function theory, one objective is to see what we can get in the quaternionic setting. The actual project - together with professor Cinzia Bisi - is about some of these possible generalizations with the aim of getting some results about interpolation problems in the quaternionic setting.

The theory of slice regular function gives some generalizations of 'complex results' but also gives rise to new phenomena. An approach to this theory (by Ghiloni and Perotti) associates such maps to 'stem functions' defined on complex domains and it is quite powerful since it leads to generalizations to real alternative $*$-algebras such as higher dimensional Clifford algebras $C\ell_{0,m}$ or octionions $\mathbb{O}$.

Previous Work

During the years of the Ph. D. course in Mathematics, the main subject of my work has been *one-parameter (continuous) semigroups of holomorphic self-maps* in the unit disk $\mathbb{D} := \{|z| < 1\}$ in the complex plane $\mathbb{C}$, given by families of maps $\{\phi_t\}_{t\geq 0}$ holomorphic from the disk to itself, such that $\phi_0 = \mathrm{id}_{\mathbb{D}}$ and $\phi_{t+s} = \phi_t \circ \phi_s$, with continuous (hence analytic) dependence on the parameter t. By complex analysis facts, these maps are forced to be univalent. This corresponds to a continuous counterpart of the problem of studying iteration of univalent maps from $\mathbb{D}$ to itself.

In particular I dealt with the case of non-elliptic semigroups, where all orbits converge to a point in the boundary. In this case, a key tool is the association with a *canonical model* (one-to-one up to translations), a conformal transformation $h: \mathbb{D} \xrightarrow{\sim} h(\mathbb{D})$ of the disk intertwining the semigroup iterate $\phi_t(z)$ with a (say vertical) translation $z \mapsto z + it$. Many dynamical properties of non-elliptic semigroups are in fact detected by the geometric properties of their Königs domains $h(\mathbb{D})$.

One goal is to find estimates about the convergence of the orbits to the limit point. By using hyperbolic (Poincaré) metrics in the disk and in simply connected domains one introduces relevant quantities named *total, orthogonal and tangential speeds of convergence*, which turn out to be useful for the scope.

A first result obtained in this sense is about an asymptotic monotonicity result for the orthogonal speed with respect to inclusions of the Königs domains.

Another result is about the faster asymptotic behavior of the tangential speed, which correspond to the case of groups of parabolic automorphisms, and more generally when the model domain is a so-called 'vertical half-sector'. It could be interesting to have a general picture of all the possible cases in which we have this extremal situation.

A recent work is about finding several conditions characterizing the semigroups of *finite shift* (or with *strong tangential convergence* to the limit point). Morally, these conditions say that the canonical model must be quite similar to the case of groups of parabolic automorphisms, where $h(\mathbb{D})$ is a vertical half-plane.

Tools that are used in the works above are hyperbolic geometry, conformal transformations, Gromov hyperbolicity (providing geodesic stability) and also harmonic measures in simply connected domains of the plane.

One of the most interesting tasks in the field is finding relevant examples with some prescribed behavior (or proving that they cannot exist). For instance, one can construct examples where we have an oscillating asymptotic behavior of the (total) speed of convergence.

As the construction of canonical holomorphic models is available also in the case of discrete iteration of univalent maps, a natural question is if some result or technique can be extended. Also the case of the unit ball in $\mathbb{C}^n$ should be interesting, but in this case much should be different, as the theory of several complex variables is different in many ways from the single variable case.

Topics of interest

Iteration of holomorphic maps and one-parameter semigroups of holomorphic maps. Canonical models.

Composition operators.

Univalent maps. Conformal transformations. Extension to the boundary.

Estimates of hyperbolic metrics.

Gromov hyperbolicity for (geodesic) metric spaces.

Potential theory (continuous and discrete setting). Harmonic measures.

Hypercomplex analysis: slice regular functions over quaternions, real Clifford algebras, octonions.

Interpolation problems.

Publications

Preprints

1. C. Bisi, D. Cordella, *Multipoint Schwarz–Pick Lemma for the quaternionic case* http://arxiv.org/abs/2312.09664

Journal Articles

1. D. Cordella, *Holomorphic semigroups of finite shift in the unit disc*, J. Math. Anal. Appl., 513(2) (2022), 126213, https://doi.org/10.1016/j.jmaa.2022.126213
2. D. Cordella, *Asymptotic upper bound for tangential speed of parabolic semigroups of holomorphic self-maps in the unit disc*, Annali di Matematica Pura e Applicata, 200(6) (2021), 2767–2784, https://doi.org/10.1007/s10231-021-01100-x

3. F. Bracci, D. Cordella, M. Kourou, *Asymptotic monotonicity of the orthogonal speed and rate of convergence for semigroups of holomorphic self-maps of the unit disc*, Rev. Mat. Iberoam. (2021), 527–546, https://doi.org/10.4171/rmi/1283

Seminars

5/7/2022 *Speeds of convergence of non-elliptic holomorphic semigroups in the unit disk* - Workshop *Holomorphic Semigroups in Rome*, Rome (Italy)

Schools/Conferences

19/9/2022-23/9/2022 CIRM Conference *Complex Analysis and Geometry XXVI*, Levico Terme (Italy)

29/8/2022-2/9/2022 INdAM Meeting *New Trends in Holomorphic Dynamics*, Cortona (Italy)

5/7/2022-8/7/2022 Workshop *Holomorphic Semigroups in Rome*, Rome (Italy)

6/9/2021-10/9/2021 INdAM Workshop *Gromov hyperbolicity and negative curvature in Complex Analysis*, Cortona (Italy)

7/6/2021-11/6/2021 CIRM-ICTP Online Meeting *Complex Analysis and Geometry XXV*

23/9/2019-27/9/2019 INdAM Workshop *(New trends in) Complex and Fourier Analysis, and Operator Theory*, Rome (Italy)

10/6/2019-14/6/2019 CIRM Conference *Complex Analysis and Geometry XXIV*, Levico Terme (Italy)

23/7/2018-17/8/2018 Summer School *Scuola Matematica Interuniversitaria 2018*, Perugia (Italy)

Teaching

1. Tutor of the course *Matematica* held by Prof. F. Radulescu, Università di Roma 'Tor Vergata', Degree in Biology (1st year), Academic year 2020-2021
2. Tutor of the course *Analisi Matematica 3* held by Prof. A. Sorrentino, Università di Roma 'Tor Vergata', Degree in Mathematics (2nd year), Academic year 2019-2020
3. Tutor of the course *Analisi Matematica 1A*, module held by Prof. N. Arcozzi, Università di Bologna, Degree in Mathematics (1st year), Academic year 2016-2017

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